Bayesian inference of the inflation and reheating model parameters

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Abstract. In this paper, we perform a Bayesian inference of the parameters of the Starobinsky inflationary model with subsequent reheating stage using the MCMC method and the observed data on the anisotropy of the cosmic microwave background collected by the Planck collaboration and on baryonic acoustic oscillations. The reheating stage is modeled by a single parameter $R_{\rm reh}$, which contains a combination of the reheating temperature $T_{\rm reh}$ and the effective equation of state during reheating $\bar{\omega}_{\rm reh}$. Using the modified Boltzmann code CLASS and the MontePython program with the GetDist package, we perform a direct analysis of the space of model parameters and obtain their posterior distributions. By computing the Kullback–Leibler divergence, we estimate the information gained by the parameter inference from the observed data. In the proposed parametrization, we achieve 7.73 bits of information about the inflaton potential amplitude and 1.64 bits of information about the reheating parameter. The results are compared with those existing in the literature implying that the specification of inflationary model allows to better constrain the reheating stage than in the model-independent approach to inflation. Finally, we draw constraints for the reheating temperature and average equation of state. Although the former can vary within 16 orders of magnitude (in the 95% credible interval), for the latter there is a clear preference for the values larger than zero, meaning that the usual dust-like equation of state $\bar{\omega}_{\rm reh} = 0$ is excluded at more than 2σ level.

Keywords: inflation; reheating; MCMC; CMB; Planck; Bayesian inference; CLASS; MontePython.

1 Introduction

Modern physicists are well aware of the picture of the evolution of the Universe, starting from the radiation dominated era (RD), during which the Robertson–Walker scale factor $a(t)$ grew according to the law $\sqrt{t}$. However, for the successful realization of the evolution of the Universe within the framework of the standard Big Bang model, physically unlikely initial conditions are imposed on the Universe at its very origin, which lead to serious cosmological problems, such as the horizon problem, the flatness problem, the entropy problem and the problem of initial inhomogeneities (for more details see [24]).

A solution to these problems is provided by the inflationary model, which was proposed by a number of authors in 1979–1981 [10, 20, 22], and developed by Alan Guth in 1981 [8]. The main idea of the inflationary model is the presence of accelerated expansion of the Universe, which took place before the RD era. At this stage, the equation of state was approaching the vacuum $p \approx -\rho$, the Universe was expanding rapidly and, finally, became homogeneous on large scales and spatially flat with high accuracy.

The most remarkable aspect of inflationary theories is that they lead to a natural quantum mechanical mechanism for the origin of cosmological fluctuations observed in cosmic microwave background (CMB), large-scale structures (galaxy clusters, voyages, etc.), and potentially in the gravitational wave background (which is predicted to be detected by the LIGO/VIRGO collaboration [15] and PTA pulsar observations [23]).

The key role in inflationary models testing is played by measurements of the anisotropy of the cosmic microwave background, starting with the first detection by the COBE satellite (NASA) [5], the measurement of Doppler peaks by the WMAP satellite [25] and ending with the latest data from the Planck mission [19]. The detection of the CMB polarization [11] was another important achievement and will be of key importance in future studies.

However, to build a complete picture of the early Universe evolution, it is also important to understand how the inflationary stage transitions into the RD era. At the end of inflation, the inflaton is usually thought to oscillate around its minimum potential, gradually decaying and transferring energy to the relativistic plasma. This post-inflationary process, which fills the Universe with ordinary matter, is known as the reheating. The traditional approach to describing the reheating is based on the fact that inflationary oscillations, which can be interpreted as a set of inflationary particles with zero momentum, cause the formation of elementary particles of the Standard Model. Interacting with each other, they come to a state of thermodynamic equilibrium, launching the standard Big Bang cosmology [24].

Usually, the literature neglects the post-inflationary reheating stage, considering it to be instantaneous. However, as shown by the authors of [18], the observed data already carry a certain amount of information about this stage (1.3 ± 0.18 bits). This means that the interpretation of the observed data without taking into account the reheating stage can lead to a distortion of the picture of the Universe evolution.

In this paper, we perform a Bayesian parameter inference of the Starobinsky inflation model with the reheating stage using the MCMC algorithm and the observed data on the anisotropy of the CMB collected by the Planck collaboration [19] and on baryon acoustic oscillations [6].

This paper is organized as follows: in Section 2, we give a brief theoretical background on inflationary theories and the reheating phase; in Section 3, we formulate the mathematical problem and outline the features of the numerical methods used; in Section 4, we present the results and corresponding plots and analyze them; in Section 5, we summarize our work. Throughout, we will work in the natural system of units $c = \hbar = k_{\mathrm{B}} = 1$ and use the Planck mass $M_{\mathrm{Pl}} = 1.22 \times 10^{19}$ GeV.

2 Literature review

2.1 Inflation model

To realize the inflationary stage, it is enough to assume the existence of a scalar field ϕ, called the inflaton, such that at some early time it takes a value at which the potential $V(\phi)$ is large but almost constant [24]. Initially, this scalar field 'slides' down with the potential very slowly, so that the Hubble parameter decreases slowly enough, and before the inflationary field changes significantly, more or less exponential inflation has already occurred in the Universe.

There are several hundred inflationary models [17], but among this variety, models with Starobinsky potential occupy a special place. The Starobinsky [22] model was one of the first models of inflation to be proposed. It is characterized by an action that contains the Einstein–Hilbert term together with an additional term that depends on the second order of scalar curvature

$$S = -\frac{M_{\mathrm{Pl}}^2}{16\pi} \int \mathrm{d}^4x \, \sqrt{-g} \left[R - \frac{R^2}{6M^2} \right],$$

where R denotes the scalar Ricci curvature, g is the determinant of the metric $g_{\mu\nu}$, and M is the free parameter of the Starobinsky model expressed in units of mass.

The Starobinsky action can be rewritten in a form that explicitly includes the scalar field action term by using a conformal metric transformation

$$g_{\mu\nu} \to \exp\left(-4\sqrt{\frac{\pi}{3}}\frac{\phi}{M_{\rm Pl}}\right) g_{\mu\nu}.$$

After that, the action can be written in the form

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2}{16\pi}R + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)\right],$$

where $V(\varphi)$ is the potential of a scalar field of the form

$$V(\phi) = V_0\left[1 - \exp\left(-4\sqrt{\frac{\pi}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2. \tag{1}$$

Here $V_0 = \frac{3}{32\pi}M_{\rm Pl}^2 M^2$. It is this parameter that we will use as a free parameter of the model for Bayesian inference.

In this paper, we study the mechanism of inflation through the Starobinsky potential. This choice is motivated by the fact that the Starobinsky inflation model arises from simple physical considerations and does not use ad-hoc parameters. It can also be shown that the Higgs inflation potential [2] (which is also a very natural candidate) in the large-coupling limit reduces to the same form [17]. Thus, using this potential, we can simultaneously investigate the two most natural models of inflation: Starobinsky and Higgs.

2.2 Reheating

The theory of inflation also requires a stage of post-inflationary reheating, during which the scalar inflationary field decays into Standard Model particles and the stage of the hot Big Bang begins.

The reheating stage can be modeled using two parameters: the reheating temperature $T_{\rm reh}$ and the effective equation of state $\bar{\omega}_{\rm reh}$, which is defined as follows [7]:

$$\bar{\omega}_{\rm reh} \equiv \frac{1}{N_{\rm reh} - N_e}\int_{N_e}^{N_{\rm reh}} \omega_{\rm reh}(N)\,\mathrm{d}N,$$

where $\omega_{\rm reh}(N) = \frac{p}{\rho}$ is the equation of state, N_e and $N_{\rm reh}$ are the number of e-folds at the end of inflation and reheating, respectively.

Next, we need to calculate the number of e-folds from the moment when the reference mode goes beyond the horizon to the end of inflation $N_* = \ln(\frac{a_e}{a_*})$. The condition for the mode with momentum k_* to go beyond the horizon is defined as $\frac{k_*}{a_* H_*} = 1$. Let us write this expression as follows:

$$\frac{k_*}{a_0}\frac{a_0}{a_{\rm reh}}\frac{a_{\rm reh}}{a_{\rm end}}\frac{a_{\rm end}}{a_*}\frac{1}{H_*} = 1.$$

According to the entropy conservation law

$$s = g_{*,0}^{(s)} a_0^3 T_0^3 = g_{*,\rm reh} a_{\rm reh}^3 T_{\rm reh}^3,$$

where s is the entropy density, g_* is the effective number of degrees of freedom, $g_*^{(s)}$ is the effective number of entropic degrees of freedom, T is the temperature, a is the scale factor.

Then,

$$\frac{a_0}{a_{\rm reh}} = \frac{T_{\rm reh}}{T_0}\left(\frac{g_{*,\rm reh}}{g_{*,0}^{(s)}}\right)^{\frac{1}{3}}.$$

Using the energy conservation law

$$\dot{\rho} + 3H(\rho + p) = 0,$$

we can relate the scaling factors and energies at the end of inflation and reheating through $\bar{\omega}_{\rm reh}$

$$\frac{a_{\rm reh}}{a_e} = \left(\frac{\rho_{\rm reh}}{\rho_e}\right)^{-\frac{1}{3(1+\bar{\omega}_{\rm reh})}}.$$

All the energy at the end of the reheating is transferred to ultra-relativistic particles, so the energy density at the end of the reheating can be written as

$$\rho_{\rm reh} = \frac{\pi^2}{30} g_{*,\rm reh} T_{\rm reh}^4.$$

So,

$$\frac{a_{\rm reh}}{a_e} = \left(\frac{\pi^2}{30} g_{*,\rm reh}\frac{T_{\rm reh}^4}{\rho_e}\right)^{-\frac{1}{3(1+\bar{\omega}_{\rm reh})}}.$$

Thus, the number of e-foldings from the moment when the reference mode goes beyond the horizon until the end of inflation is determined by the following expression

$$N_* = \ln\left[\frac{H_*}{M_{\rm Pl}}\frac{a_0 M_{Pl}}{k_*}\frac{T_0}{T_{\rm reh}}\left(\frac{g_{*,\rm reh}}{g_{*,0}^{(s)}}\right)^{\frac{1}{3}}\left(\frac{\pi^2}{30} g_{*,\rm reh}\frac{T_{\rm reh}^4}{\rho_e}\right)^{\frac{1}{3(1+\bar{\omega}_{\rm reh})}}\right].$$

As shown by the authors of [18], for a complete description of the reheating process, it is sufficient to consider a certain combination of the parameters $T_{\rm reh}$ and $\bar{\omega}_{\rm reh}$, rather than each of them separately. Therefore, we will use the so-called rescaled reheating parameter

$$R_{\rm reh} = \frac{a_e}{a_{\rm reh}}\left(\frac{\rho_e}{\rho_{\rm reh}}\right)^{\frac{1}{4}}\frac{\rho_e^{\frac{1}{4}}}{M_{\rm Pl}} = \left(\frac{\pi^2}{30} g_{*,\rm reh}\frac{T_{\rm reh}^4}{\rho_e}\right)^{\frac{1}{3(1+\bar{\omega}_{\rm reh})}}\frac{\rho_e^{\frac{1}{2}}}{\left(\frac{\pi^2}{30} g_{*,\rm reh} T_{\rm reh}^4\right)^{\frac{1}{4}} M_{\rm Pl}}. \tag{2}$$

Finally, using the parameter $R_{\rm reh}$ and the Friedman equation $\rho_e = \frac{3}{8\pi} H_e^2 M_{Pl}^2$, we obtain the expression for determining the required number of e-folds

$$N_* = N_0 + \ln R_{\rm reh} + \ln\left(\frac{H(N_*)}{H_e}\right), \tag{3}$$

where

$$N_0 = \ln\left(\frac{T_0 a_0}{k_*}\frac{(g_{0,*}^{(s)})^{\frac{1}{3}}}{g_{\rm reh,*}^{\frac{1}{12}}}\frac{\pi\sqrt{8}}{\sqrt{3}\sqrt[4]{30}}\right) \approx 62.7.$$

This expression is an implicit equation in N_*, the solution of which depends on the dynamics of the inflaton field. The equation (3) allows us to obtain the exact value of the number of e-foldings between the moment when the reference mode goes beyond the horizon and the end of inflation.

2.3 Cosmic microwave background

As noted above, while we do not yet have reliable observational evidence for the inflationary model, we do have cosmic sources of information that can impose constraints on inflation and reheating models. The most important source of information is the relic radiation, or cosmic microwave background, the anisotropy of which is studied by the Planck collaboration (ESA mission).

The CMB is a residual electromagnetic radiation that originated about 380 000 years after the Big Bang starts, during the recombination epoch, when the temperature of the Universe dropped enough to form neutral hydrogen atoms, causing photons to stop interacting with matter and start to spread freely in space. Measurements of the CMB show that it corresponds to the spectrum of an ideal blackbody with a temperature of $T_0 \approx 2.725$ K, demonstrating high isotropy with fluctuations at the level of $\frac{\delta T}{T} \sim 10^{-5}$. However, it is this small anisotropy that is of the greatest scientific importance, as it constrains the parameters of inflation, the properties of decaying or annihilating particles, primary black holes, topological defects, primary magnetic fields and other exotic physics.

For the quantitative analysis, the Planck collaboration [19] introduces two-point angular correlation functions and performs a harmonic expansion of the CMB map. Letting T, Q and U denote the intensity and Stokes parameters for the polarization, we define

$$
\begin{aligned}
a_{\ell m} &= \int \mathrm{d}\hat{n}\, Y^*_{\ell m}(\hat{n}) T(\hat{n}), \\
a_{E\ell m} \pm i a_{B\ell m} &= \int \mathrm{d}\hat{n}\, {}_{\pm 2}Y^*_{\ell m}(\hat{n})(Q \pm iU)(\hat{n}),
\end{aligned}
$$

where ${}_{\pm 2}Y_{\ell m}$ are spherical spin harmonics proportional to the Wigner functions.

For the case of statistical isotropy, it is necessary that the quantities $\langle a^*_{\ell m} a_{\ell' m'} \rangle$ should be diagonal and depend only on ℓ. Then we write

$$
\langle a^X_{\ell m} a^Y_{\ell' m'} \rangle = C^{XY}_\ell \delta_{\ell' \ell} \delta_{m' m},
$$

where $X, Y \in \{T, E, B\}$ denote the temperature and polarization modes, and C^{XY}_ℓ are the angular power spectra. It is also convenient to determine the angular power spectra

$$
D^{XY}_\ell = \frac{\ell(\ell+1) C^{XY}_\ell}{2\pi}. \tag{4}
$$

The autospectrum D^{XX}_ℓ shows the approximate contribution of the logarithmic interval of the multipoles centered on ℓ to the fluctuation variance. It thus reflects the relative importance of different contributions to the signal as a function of scale.

It is the angular power spectra that carry information about the anisotropy of the relic radiation, and it is these that we will use as data for the Bayesian inference.

2.4 Bayesian inference

Bayesian inference is a method of statistical inference based on the use of Bayes theorem to estimate model parameters. The basic idea is to update the probabilities of the model parameters based on new experimental data. Bayes theorem defines the relationship between the a posteriori probability of the parameters θ given the available data D, the likelihood and the a priori distribution:

$$
P(\theta \mid D) = \frac{P(D \mid \theta) P(\theta)}{P(D)},
$$

where

- $P(\theta \mid D)$ is a posteriori distribution of the model parameters (the probability of the parameters θ after taking into account the data D);
- $P(D \mid \theta)$ is the likelihood (probability of obtaining the observed data D with the given parameters θ);
- $P(\theta)$ is a priori distribution of parameters (information about the parameters before obtaining new data);
- $P(D)$ is evidence, which acts as a normalization factor and is calculated as an integral of the likelihood over all possible values of the parameters:

$$P(D) = \int P(D \mid \theta) P(\theta)\, \mathrm{d}\theta.$$

The Bayesian approach allows not only to estimate the values of the parameters, but also to calculate the uncertainty in their values in the form of a complete posterior distribution. However, for complex models with many parameters, finding the integral $P(D)$ is usually a computationally expensive task.

To avoid calculating this integral, the Markov chain Monte Carlo (MCMC) method is used. MCMC allows you to generate a set of random samples from a posterior distribution without having to calculate the proof $P(D)$.

The MCMC algorithm builds a Markov chain – a sequence of parameters values $\theta_1, \theta_2, \theta_3, \ldots$ – such that each new state depends only on the previous one. In the equilibrium state, the density of points in the parameter space which belong to the chain approaches the posterior distribution $P(\theta \mid D)$.

The most common implementation of the MCMC method is the Metropolis–Hastings algorithm. At each step, a new value of θ' is generated from the propositional distribution $q(\theta' \mid \theta)$, and its acceptance is performed with probability

$$A = \min\left(1, \frac{P(D \mid \theta') P(\theta') q(\theta \mid \theta')}{P(D \mid \theta) P(\theta) q(\theta' \mid \theta)}\right).$$

If the new value is accepted, then $\theta_{n+1} = \theta'$, otherwise $\theta_{n+1} = \theta_n$. For a more detailed introduction to the MCMC method, we refer the reader to [21].

3 Aims and numerical methods review

We aim to study the parameter space of the Starobinsky inflation model (1) with a reheating $\theta = (V_0, R_{\rm reh})$ using the MCMC method. Unlike the authors of [18], who use a model-independent approach to inflation stage, we apply the MCMC algorithm directly to the parameters V_0, $R_{\rm reh}$. We compare the posterior distributions obtained with their results. And we will conclude whether this approach gives results different from the inflation model-independent approach and whether it is worth using.

To do this, we use the Boltzmann code CLASS [3,13], which solves all the Universe background dynamics, and the MontePython program [1,4], which implements the MCMC algorithm. We analyze the resulting chains using the capabilities of the GetDist package [14].

By default, CLASS does not take into account the reheating stage. Therefore, we modified the CLASS code by implementing the reheating stage in the `primordial` module by the equation (3).

The data against which we will compare our model are the angular spectra (4) of temperature (TT), polarization (TE and EE) and lensing ($\phi\phi$), which are contained in the likelihood functions 'Planck_highl_TTTEEE', 'Planck_lowl_EE', 'Planck_lowl_TT', 'Planck_lensing' [19], and the baryon acoustic oscillation observations (BAO) [6] contained in the likelihood functions 'bao_boss_dr12', 'bao_smallz_2014'.

As a priori distributions, we take homogeneous distributions on the logarithmic scale, including a wide range of values of $\ln R_{\rm reh} \in \left[-46.15 + \frac{1}{3}\ln\left(\frac{\rho_{\rm end}}{M_{\rm Pl}^4}\right)\right]$ and $\ln V_0 \in [-150, -28]$ in Planck masses. These values are due to the fact that the reheating that occurs after inflation requires that $\rho_{\rm reh} \leq \rho_{\rm end}$, while we expect the average equation of state of the Universe during this period to satisfy the condition $-1/3 < \bar{\omega}_{\rm reh} < 1$, where the lower bound guarantees that inflation has stopped. The requirement to avoid disrupting Big Bang nucleosynthesis imposes the constraint $\rho_{\rm reh} > \rho_{\rm nuc}$, and we set the lower bound at $\rho_{\rm nuc} = g_{*,\rm nuc}\frac{\pi^2}{30}T_{\rm nuc}^4$ with $T_{\rm nuc} = 10\,{\rm MeV}$. The equation (2) and the condition $\rho_{\rm end} < M_{\rm Pl}^4$ give the widest possible a priori distribution for the reheating parameter $\ln R_{\rm reh} = \left[-46, 15 + \frac{1}{3}\ln\left(\frac{\rho_{\rm end}}{M_{\rm Pl}^4}\right)\right]$ [18]. The choice of the upper limit for $\ln V_0 = -28$ is due to the fact that at higher values of the parameter, the agreement with the Planck Collaboration data becomes impossible.

In addition, the Planck Collaboration likelihood functions require consideration of 21 'nuisance' parameters, which have a certain impact on the rate of chain convergence. We choose the standard distributions for these parameters proposed by the Planck Collaboration as a priori distributions.

4 Results and discussion

As a result of the analysis of Markov chains of five million points, we obtained posterior distributions for the parameters $\ln R_{\rm reh}$ and $\ln V_0$, which are shown in Figure 1. Also, the best fit values, mean values with standard deviation, and 95% credible limits for $\ln R_{\rm reh}$ and $\ln V_0$ are shown in Table 1.

Param	best-fit	mean$\pm\sigma$	95% lower	95% upper
$\ln R_{\rm reh}$	-6.8916	$-3.077^{+4.6}_{-4.3}$	-10.9	5.4
$\ln \frac{V_0}{M_{\rm Pl}^4}$	-29.34	$-29.47^{+0.14}_{-0.15}$	-29.7	-29.2

Table 1. Best fit values, mean values with standard deviation and 95% credible limits for $\ln R_{\rm reh}$ and $\ln V_0$. The likelihood function was maximized to $-\ln\mathcal{L}_{\rm min} = 1402.57$, which corresponds to the minimum value for the function $\chi^2 = 2805.14$.

Figure 1 shows that within $\ln V_0 \in [-29.8, -29.1]$ the parameters $\ln R_{\rm reh}$ and $\ln V_0$ are linearly dependent, the calculated correlation coefficient reaches -1, which indicates that the contribution of the parameters $\ln R_{\rm reh}$ and $\ln V_0$ to the dynamics of the Universe expansion is inversely proportional within these limits.

Also, from Table 1 we can conclude that the observed data give rather strict constraints on the parameter $\ln V_0$, but the constraints on the parameter $\ln R_{\rm reh}$ are very weak: $R_{\rm reh}$ can take values within 8 orders of magnitude.

We can estimate the amount of information about the parameters of our model provided by the observed data using the Kullback–Leibler divergence [12]:

$$D_{\rm KL}^{\rm reh} = \int P(\ln R_{\rm reh} \mid D)\ln\frac{P(\ln R_{\rm reh}|D)}{P(\ln R_{\rm reh})}\,{\rm d}\ln R_{\rm reh},$$

$$D_{\rm KL}^{\rm inf} = \int P(\ln V_0 \mid D)\ln\frac{P(\ln V_0|D)}{P(\ln V_0)}\,{\rm d}\ln V_0.$$

These values are a measure of how much the obtained posterior distributions differ from the corresponding a priori distributions. For our model, we calculated the values of $D_{\rm KL}^{\rm reh} \approx 1.64$,

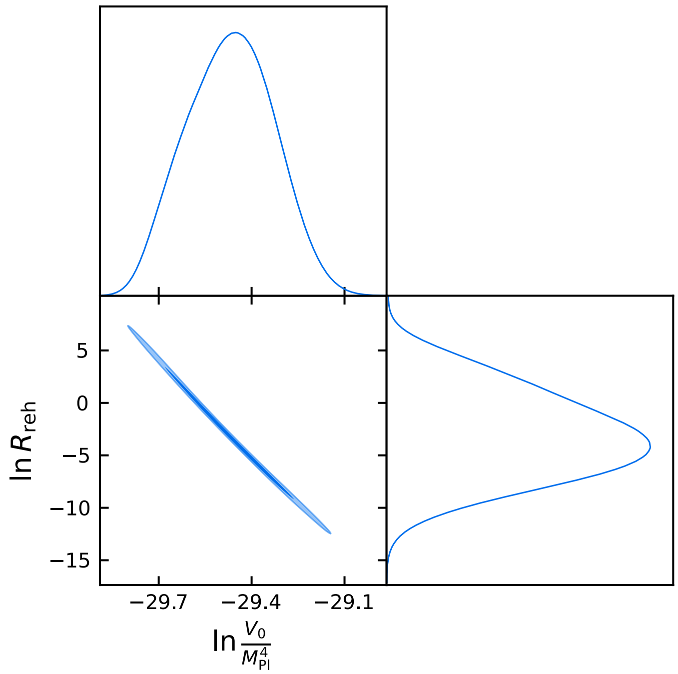

Figure 1. The two-dimensional posterior distribution for the parameters $\ln R_{\rm reh}$ and $\ln V_0$ obtained by statistical analysis of Planck 2018 and BAO data. The light blue color indicates the 95% credible region, and the blue color indicates the 68% credible region.

$D_{\rm KL}^{\rm inf} \approx 7.73$ in bits. Thus, although the data contain much more information about the inflation parameter V_0, the value of $D_{\rm KL}^{\rm reh} \approx 1.64$ is already large enough to conclude that it is necessary to take into account the model of non-instantaneous reheating when considering the evolution of the Universe, which is consistent with the results of the authors of [18]. However, by applying the MCMC algorithm directly to the model parameters, we were able to obtain more information about the reheating parameter $R_{\rm reh}$ than they did using a model-independent approach. They obtained the result $D_{\rm KL}^{\rm reh} \approx 1.35$.

From the obtained posterior distribution of the parameter $\ln R_{\rm reh}$, we can calculate the distributions for the parameters $\ln T_{\rm reh}$ and $\bar{\omega}_{\rm reh}$ by performing a much less computationally expensive MCMC algorithm. The results are shown in Table 2.

Param	best-fit	mean$\pm\sigma$	95% lower	95% upper
$\ln \frac{T_{\rm reh}}{M_{\rm Pl}}$	-44.6460	$-30.861^{+13.0}_{-12.4}$	-50.1	-10.6
$\omega_{\rm reh}$	0.6715	$0.579^{+0.2}_{-0.2}$	0.0394	0.963
$\ln \frac{V_0}{M_{\rm Pl}^4}$	-29.56	$-29.43^{+0.11}_{-0.11}$	-29.7	-29.2

Table 2. Best fit values, means with standard deviation, and 95% limits for $\ln \frac{T_{\rm reh}}{M_{\rm Pl}}$, $\bar{\omega}_{\rm reh}$, and $\ln \frac{V_0}{M_{\rm Pl}^4}$.

We obtain that the reheating temperature can vary within 16 orders of magnitude (in the 95% credible interval), but for average equation of state there is a clear preference for the values larger than zero, meaning that the usual dust-like equation of state $\bar{\omega}_{\rm reh} = 0$ is excluded at more than 2σ level.

5 Conclusions

Before the RD era, the evolution of the early Universe is defined by an inflationary phase and a reheating phase. The physics of inflation has been actively studied and is already tightly constrained by observations such as the CMB anisotropy, but the reheating is not always taken into account as a key transition stage between inflation and the RD era. In this paper, we investigate the constraints on the parameters of the Starobinsky potential model with subsequent reheating imposed by the observed data on the CMB anisotropy and baryonic acoustic oscillations.

The observed data strictly constrain the potential parameter $\ln V_0$, since the mean value and 95% credible interval are small $\left(-29.47^{+0.15}_{-0.15}\right)$. On the contrary, for $\ln R_{\rm reh}$ the constraints are much weaker, which means that a wide range of values of the reheating parameter is possible, covering about 8 orders of magnitude.

The value of the Kullbak–Leibler divergence for $\ln R_{\rm reh}$ is $D^{\rm reh}_{\rm KL} \approx 1.64$ bits. This means that although the constraints on this parameter are weak, the data obtained still contain enough information to confirm the need to take into account the non-instantaneous reheating stage in cosmological models.

The obtained divergence value of $D^{\rm reh}_{\rm KL} \approx 1.64$ bits is higher than in previous studies where a model-independent approach to inflation was used ($D^{\rm reh}_{\rm KL} \approx 1.35$) [18]. This indicates that MCMC estimation directly on the model parameters can provide more information about the parameter $R_{\rm reh}$.

At the same time, for the reheating temperature $\ln T_{\rm reh}$, the 95% credible interval covers a huge range (from -49.1 to -10.6), which indicates a serious uncertainty in this parameter. The average value of the effective state parameter during reheating $\bar{\omega}_{\rm reh} = 0.591$ indicates that reheating probably occurred in a medium with a tighter pressure-energy density dependence than for conventional radiation ($\omega_{\rm reh} = 1/3$). However, the 95% credible limits for $\bar{\omega}_{\rm reh}$ also cover a fairly wide range (from 0.0394 to 0.966), which indicates some uncertainty in the physical conditions of the reheating stage.

Thus, our results confirm the importance of taking the reheating stage into account in cosmological models. Despite the fact that the observed data do not yet provide strict constraints on the reheating parameters, the influence of this stage cannot be neglected. From the Planck 2018 and BAO data, it is already possible to obtain approximately 1.64 bits of information about the reheating parameter $\ln R_{\rm reh}$. In addition, there are indications that directly applying the MCMC method directly to the model parameters allows us to obtain more information compared to inflation model-independent approaches.

Note added: While finalizing this work, new data from the Atacama cosmology telescope (ACT) [16] was released, providing important constraints relevant to Starobinsky inflation. Motivated by this, we performed a Bayesian analysis of both Starobinsky and Higgs inflation models including the reheating phase, incorporating the ACT observational data. The results of this analysis are presented in [9].

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Оцінка параметрів моделі інфляції та розігріву методами баєсівської статистики

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Анотація. У даній роботі проведено баєсівський аналіз параметрів моделі інфляції Старобінського з розігрівом з використанням методу MCMC і спостережними даними про анізотропію реліктового випромінювання, зібраними колаборацією Planck, і про баріонні акустичні осциляції. Стадію розігріву змодельовано одним параметром R_{reh}, який містить у собі комбінацію температури розігріву T_{reh} та ефективного рівняння стану речовини впродовж розігріву $\bar{\omega}_{\mathrm{reh}}$. За допомогою модифікованого больцманівського коду CLASS і програми MontePython з пакетом GetDist зроблено прямий аналіз простору параметрів моделі та отримано їх постеріорні розподіли. За допомогою дивергенції Кульбака–Ляйблера, оцінено кількість інформації, отриманої в результаті аналізу параметрів зі спостережних даних. У запропонованій параметризації отримано 7.73 біта інформації про амплітуду потенціалу інфлатона та 1.64 біта інформації про параметр розігріву. Отримані результати порівняно з тими, що вже є в літературі, і вони вказують на те, що специфікація моделі інфляції дозволяє краще обмежити етап розігріву, ніж у модельно-незалежному підході до інфляції. Нарешті, встановлено обмеження на температуру розігріву та середнє рівняння стану. Хоча перша може змінюватися в межах 16 порядків величини (у 95% довірчому інтервалі), для другого спостерігається чітка перевага значень, більших за нуль, що означає, що звичайне рівняння стану пилу $\bar{\omega}_{\mathrm{reh}} = 0$ виключається з більш ніж 2σ рівнем значущості.

Ключові слова: інфляція; розігрів; MCMC; реліктове випромінювання; Planck; баєсівський аналіз; CLASS; MontePython.